Closing Tues: 2.8
Closing Thurs: 3.1-2
Closing Fri: 3.3

Exam 1 is Tues, Oct. 17 in your normal quiz section. Covers: 2.1-3,2.5-8, 3.1-3.

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
3. $\frac{d}{dx}\left(cf(x)\right) = cf'(x)$
4. $\frac{d}{dx}(x^n) = nx^{n-1}$

Entry Task: Find the derivatives of

$a)\ g(x) = \frac{x^3}{2} - \frac{3}{\sqrt{x^5}}$

$b)\ f(x) = \frac{20}{3}x^3 - \frac{7x^2}{2} - 6x + 90$

$c)$ Find all *x* at which $y = f(x)$ has a horizontal tangent.

Application Notes:

1. $f'(a)$ = "slope of tangent to f(x) at *a*"
2. *Tangent Line Equation*:

$$y = f'(a)(x - a) + f(a)$$

3. $-\frac{1}{f'(a)}$ = "slope of *normal* to f(x) at *a*"
4. *Normal Line Equation*:

$$y = -\frac{1}{f'(a)}(x - a) + f(a)$$

Example: Let $f(x) = x^2 + 3$.

a) Find $f'(x)$
b) Find the equations for the tangent and normal lines at x = 2.
c) Find all points on y = f(x) at which the normal line would also pass through (0,10)

Finishing 3.1 (Another Rule)

5. $\dfrac{d}{dx}(e^x) = e^x$ and $\dfrac{d}{dx}(a^x) = a^x \ln(a)$

"Proof"

5. Exponential Function Rule:

For $f(x) = a^x$,

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$

$$= a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$

$$\text{Note: } \ln(a) = \lim_{h\to 0} \frac{a^h - 1}{h}$$

Find the derivative

1. $y = 5x + 3x^3 + 7e^x$

2. $y = \frac{4(2)^x}{3} - 11 + \frac{8\sqrt[3]{x^2}}{5}$

3.2 Product and Quotient Rules

6. $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$

7. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

Examples: Find y'

$a)\ y = x^3e^x$

$b)\ y = \frac{2x^4}{x^2 - 3}$

6. Product Rule Proof:

$$\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}f(x+h)\frac{g(x+h)-g(x)}{h}+g(x)\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

You do: Find $\frac{dy}{dx}$

$a)\ y=x^6e^x-\frac{5}{\sqrt{x^3}}$

$b)\ y=(x^3+1)^2+\frac{x^5}{1+e^x}$

$c)\ y = \dfrac{2x^2 + 1}{x^3 e^x}$

$d)\ y = (x^2 + 3)\sqrt{x}\, e^x$

3.3 Derivatives of Trig Functions

First a review: you will need to know all the following well in Math 124/5/6.

1. Triangle definitions

$\sin(x) = \dfrac{\text{opp}}{\text{hyp}}$	$\cos(x) = \dfrac{\text{adj}}{\text{hyp}}$
$\tan(x) = \dfrac{\text{opp}}{\text{adj}}$	$\cot(x) = \dfrac{\text{adj}}{\text{opp}}$
$\sec(x) = \dfrac{\text{hyp}}{\text{adj}}$	$\csc(x) = \dfrac{\text{hyp}}{\text{opp}}$

Thus,

$\sec(x) = \dfrac{1}{\cos(x)}$	$\csc(x) = \dfrac{1}{\sin(x)}$
$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$	$\cot(\text{x}) = \dfrac{\cos(x)}{\sin(x)}$

2. **Know what their graphs look like**.
3. **Know their inverses** and how to use them (and how to get more solutions)

4. Know the standard values (unit circle) and circular motion

Examples (do NOT use a calculator)

$$\cos\left(\frac{\pi}{6}\right) =$$

$$\sec\left(-\frac{\pi}{4}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) =$$

$$\sin^{-1}\left(\frac{1}{2}\right) =$$

$$\tan^{-1}(1) =$$

5. Know the main identities.

$$\sin^2(x) + \cos^2(x) = 1$$

$$2\sin(x)\cos(x) = \sin(2x)$$